AH-1546-CV-19-S

M.Sc. (Final) MATHEMATICS

Term End Examination, 2019-20

PARTIAL DIFFERENTIAL EQUATIONS, MECHANICS AND GRAVITATION

Time : Three Hours] **[Maximum Marks : 100**

Note : Attempt any five questions. All questions carry equal marks.

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1. (a) Prove that
$$\int_0^\infty \frac{\sin^2 t}{t^2}\, dt = \frac{\pi}{2}$$
(b) Find
$$L^{-1}\left\{\frac{3p+1}{(p-2)(p^2+1)}\right\}$$

2. (a) Solve
$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$
(b) Solve
$$p^2 + q^2 - 2px - 2qy + 1 = 0$$

3. (a) Using Laplace Transform solve the differential eqution
$$\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial x^2}, \quad \text{where}$$
$$y(0,t) = y(s,t) = 0 \ and \ y(x,0) = 10\sin 4x$$
(b) Derive the one-dimensional wave equation.

4. (a) Derive the fundamental solution of Heat equation.
(b) Find the shortest distance between two points in a plane.

5. (a) Define Ruthian function and find the Ruthain equations.
(b) Find the equation of motion of one dimensional harmonic oscillator using Hamilton's principle.

6. (a) Show that the transformation
$$P = \frac{1}{2}(p^2+q^2), \quad Q = \tan^{-1}\left(\frac{q}{p}\right) \text{ is canonical.}$$
(b) Show that Poisson's brackets are invariant under a canonical transformation that is show that
$$[X,Y]q,p = [X,Y]Q,p$$

7. State and prove Liouville's Theorem.

8. (a) Find the attraction of uniform sphere at external and internal point.
(b) State and prove Laplace equations.

9. (a) State and prove Gauss Theorem.
(b) Find the potential of a circular uniform disc of radius a, small thickness K and density p at an external point P on its distant q from the centre N.

10. Prove that a solid uniform hemisphere of radius a exerts no resultant attraction at a point on its axis at a distance c from the centre given by the equation
$$12c^4 - 8a^3c - 3a^4 = 0$$